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BRIDGE MONITORING BY HORIZONTAL DISPLACEMENT AT GIRDER ENDS

MONITOROWANIE MOSTÓW Z WYKORZYSTANIEM PRZEMIESZCZEŃ POZIOMYCH NA KOŃCACH UKŁADU NOŚNEGO

Abstract Vertical deflection in response to a live load is often used as an integrated indicator when diagnosing the soundness of girder bridge structures. Herein, we show how vertical deflection of a girder can be calculated by measuring the horizontal displacement generated at the ends of the girder. The horizontal displacement generated at a girder end can then be used as a new indicator in place of measuring vertical deflection at the span center. Additionally, we show that the horizontal force acting on aged bearings can be calculated from measured horizontal displacement at the upper and lower ends of a girder, allowing the sliding function of such bearings to be monitored and evaluated. In the near future, horizontal displacement at girder ends will be routinely measured in the course of bridge health monitoring.

Streszczenie Przemieszczenia pionowe spowodowane obciążeniem użytkowym są często traktowane jako zintegrowany wskaźnik jakości konstrukcji. W niniejszej pracy pokazano, jak można ustalić wielkość przemieszczeń pionowych na podstawie pomiaru przemieszczeń poziomych występujących na końcach układu nośnego. Przemieszczenia poziome występujące na końcach układu nośnego mogą być traktowane jako nowy wskaźnik jakości konstrukcji zamiast mierzenia przemieszczeń pionowych w środku rozpiętości przęseł. Ponadto pokazano, że siła pozioma działająca na skrajne łożysko może być wyznaczona na odstawie pomiaru przemieszczeń poziomych górnej i dolnej krawędzi dźwigara, pozwalając na monitorowanie i wyznaczanie funkcji przesuwu takich łożysk. W bliskiej przyszłości przemieszczenia poziome na końcach dźwigarów będą rutynowo mierzone w celu monitorowania jakości mostów.

1. Introduction

Structural health monitoring techniques are systems consisting of monitoring techniques and soundness diagnostics. For a large structure such as a bridge, important challenges include the choice of an appropriate monitoring location, and which characteristics to monitor.

Vertical deflections in response to live loads are often used as an integrated indicator when diagnosing structural soundness because excessive increases in deflection may be due to girder corrosion and erosion of material, the decline of elastic coefficients over time, stiffness degradations of main girders due to joint defects, and other pertinent causes. Vertical deflections in a bridge are generally measured to evaluate structural soundness, but measuring such deflections is often difficult, especially near the span center, because bridges typically span long distances, such as across a river, which makes fixed point location problematic.

When trains pass along a bridge, each girder bends in response to the changing positions of the train axles on each girder, and horizontal displacements at the girder ends occur according to the deflection angle generated by vertical deflections. For a given bridge type, such as simple girder or continuous girder, vertical deflections at the span center are proportionately linked with horizontal displacements at the girder ends. Unlike vertical displacements at span centers, horizontal displacements generated at the end of a girder can be easily measured using precision displacement sensors.

In this paper, we show how the horizontal displacement generated at girder ends can be used as a new indicator in place of using vertical deflection at the center of a span. Additionally, we show that the horizontal force acting on a bearing can be calculated from the horizontal displacement observed at the upper and lower position of girder ends, and describe how the sliding function of bearings can be evaluated by monitoring horizontal displacements.

Furthermore, horizontal displacements at girder ends are observed in the field and then compared with calculated displacements. Measurements carried out before, during, and after the repair of girder bearings can be used to estimate their capability and performance before and after repair work.

2. Relation between vertical and horizontal deflection

When a concentrated load W acts at the center of a girder span, as shown schematically in Figure 1, the maximum vertical deflection δv at the span center and the deflection angle i at the girder end are obtained as shown in Eq. (1) and (2), respectively.
Note that the shear deformation of the girder is not considered in the deflection.

$$\delta_v = WL^3/\ 48\ EI = 0.02083\ WL^3\ /\ EI \tag{1}$$

$$i = WL^2\ /\ 16\ EI = 0.06250\ WL^2\ /\ EI \qquad 2)$$

where L indicates the span length of the girder, E is the elastic coefficient, and I is the area moment of inertia. The bearings of the simple girder model are assumed to be undifferentiated between fixed and movable, and the girder model is deformed symmetrically by loading. The upper and lower ends of the girder are equally displaced relative to one another in opposite directions, so the absolute values of the displacements are equal because the girder has its neutral axis along the midpoint of its cross section. The absolute value δ_h is obtained as follows, where α represents the ratio between the span length L and the height h ($\alpha = L/h$).

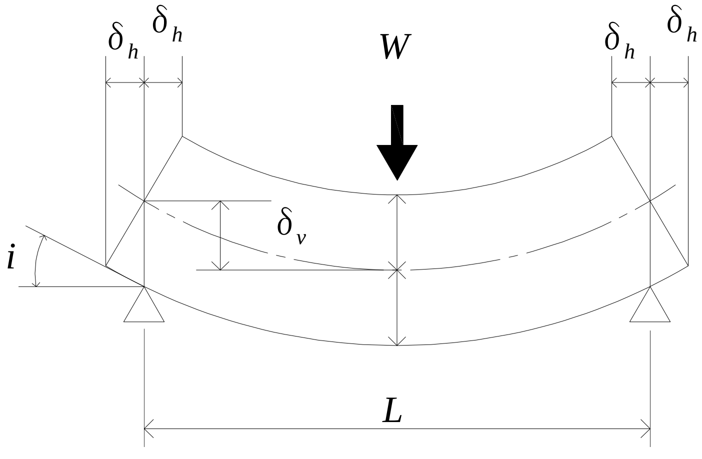

Figure 1. Deflection at girder center and girder end

$$\delta_h = (h/2) = 1.5\delta_v(h/L) = 1.5\delta_v/\alpha \tag{3}$$

The ratio between the horizontal displacement at the girder end and the vertical deflection at the span center is inversely proportional to the ratio α. When $\alpha = 20$, δh is equal to δv/13.3. Thus, the horizontal displacement generated at the girder end needs to be measured with a device that can provide precision that is roughly an order of magnitude higher than that required for measuring vertical deflection at the center of a span.

3. Relation between neutral axis length and horizontal displacement

The neutral axis length of a girder is assumed to neither expand nor contract axially when the girder deflects under load, and the neutral axis of a girder is assumed to lie along the middle of its cross-section. However, when the girder is deformed, the length of the neutral axis becomes slightly longer than the span length L because the neutral axis assumes the shape of a deflection curve. When a concentrated load *W* acts at the span center of a girder, the vertical deflection Y forms a cubic curve described by Eq. (16)

$$Y = W(3L^2X - X^3) / 48EI \ (0 \leqq X \leqq L/2) \tag{4}$$

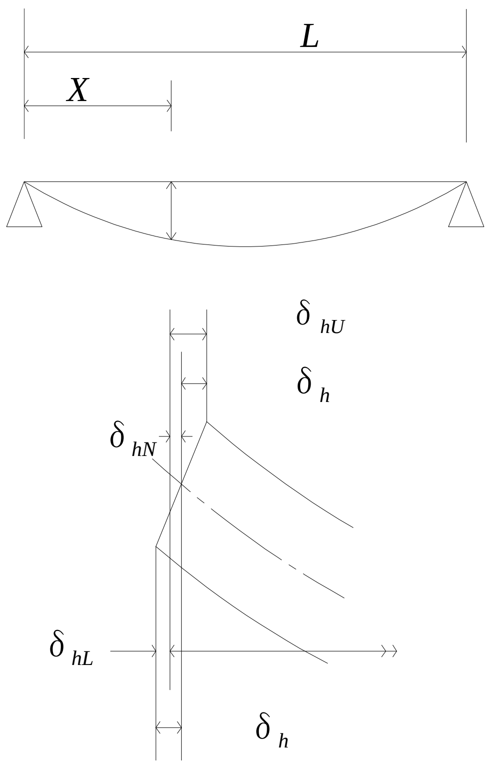

Figure 2. Deflection at girder end

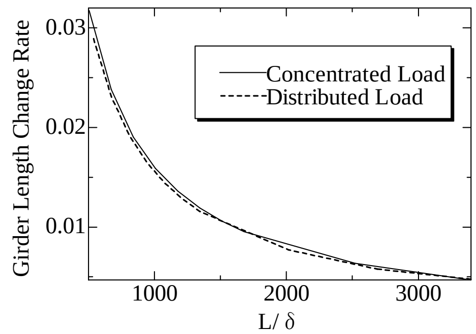

Figure 3. Change rate of girder length

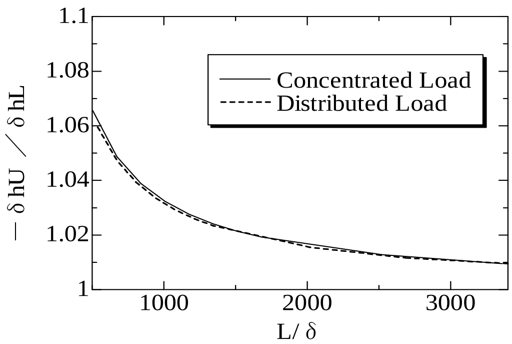

Figure 4. Horizontal deflections at girder. end

where X indicates the distance from the left support. The neutral axis length of a girder is calculated by integrating the deflection curve of the girder from $X = 0$ to $X = L$, and the length when deflected is equal to $L + 2\delta_{hN}$.

The x-axis length of the girder is reduced by the amount $2\delta h_N$ by the bending deformation because the neutral axis length *L* is constant, as shown in Figure 2. Figure 3 shows the relationship between the ratio of the deflection δ_v to the span length *L*, and the ratio of decrease in the horizontal length of the neutral axis, when the span length is 20 m, the bending moment is 7500 Nm, and h/L=1/20. The deflection at the span center is inversely proportional to the area moment of inertia under a constant bending moment. The horizontal length of the neutral axis decreases according to the deflection δ_v.

The decrease ratio of the horizontal length of the neutral axis increases when the ratio of the deflection δ_v to the span length L decreases, regardless of whether the load is concentrated or distributed. For a bridge that is easily deformed by a live load, such as a railway bridge, the horizontal length of the neutral axis is reduced by approximately 1% when the ratio of the span length to the deflection L/δ_v is 1,500. The decrease in the horizontal length of the neutral axis affects the horizontal displacement at the girder ends, and respective horizontal displacements δ_{hU} and δ_{hL} for the upper and lower ends of a girder are as follows.

$$\delta_{hU} = \delta_h + \delta_{hN}; \quad \delta_{hL} = -\delta_h + \delta_{hN} \tag{5}$$

where δ_{hN} indicates the decrease in the horizontal length of the neutral axis. The horizontal displacements δ_{hU} and δ_{hL} are positive when the girder contracts axially. Figure 4 shows the relationship between the ratio L/δ_v and the ratio $-\delta_{hU}/\delta_{hL}$. The ratio $-\delta_{hU}/\delta_{hL}$ approaches a value of 1 as the vertical deflection approaches zero. The upper end of the girder is displaced approximately 2% more than the lower end when L/δ_v = 1,500. These characteristics are essentially independent of the load type, concentrated or distributed.

4. Restraint of horizontal displacement

One consequence of the deterioration in the sliding function of a bridge bearing due to age is that the horizontal displacement at the girder end is restrained and horizontal forces acting upon the bearing are increased. Here, the substructure is assumed not to be deformed by horizontal forces, i.e., the substructure is assumed to be absolutely rigid. Hereinafter, the horizontal displacement at the girder end is estimated under this condition.

Due to deterioration in the sliding function of bearings as they age, the horizontal displacement at girder ends may be restrained and binding forces can act on bearings so that they rotate toward the bridge axis. Here, the binding force acting on a bearing is estimated when a concentrated load acts at the center of a simple girder.

The bending moment M_{RH} generated by binding force R_H at the bearing is equal to $R_H(h/2)$. When bending moment M_{RH} acts at both ends of a girder, the deflection angle i and the horizontal displacement δM_{RH} at the girder end are obtained as follows.

$$i = M_{RH} L/2\,EI \tag{6}$$

$$\delta M_{RH} = (h/2)\, M_{RH}\, L/2\,EI \tag{7}$$

Binding force R_H acts as a compressive force along the bridge axis and the horizontal displacement ΔL generated by this force is obtained as follows.

$$\Delta L = (L/2)\, R_H/EA \tag{8}$$

where A and E indicate the sectional area and the elastic coefficient of the girder. When both lower ends of a girder are fixed, the horizontal displacement generated at a girder end in response to a vertical deflection as shown in Eq. (3) is equal to the sum of δM_{RH} and ΔL, as shown in Eq. (9).

$$\delta_h = WL^2\, h/32\, EI = \delta M_{RH} + \Delta L = L\, h^2\, R_H\, (1+\beta/2) / 8\, EI \tag{9}$$

The binding force RH is obtained from Eq. (10).

$$R_H = (L/h)\, W / \{\, 2(2+\beta)\, \} \tag{10}$$

where the relationship between the cross-sectional area of the girder and the area moment of inertia is defined as follows.

$$A = (1 / \beta)\, (8\, I / h2) \tag{11}$$

The value of β is close to 1 for the cross-section area of a typical girder. When $\beta = 2$, a girder would have no cross-sectional area, and the girder shown in the left side of Figure 5 has a rectangular cross-section with $\beta = 2/3$. The values of R_H, δM_{RH} and ΔL are obtained using α and β as follows.

$$R_H = \alpha W / \{2(\beta+2)\}; \quad \delta M_{RH} = 2\delta_h / (\beta+2); \quad \Delta L = \delta_h\, \beta / (\beta+2) \tag{12}$$

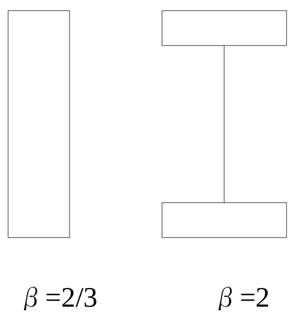

Figure 5. β value

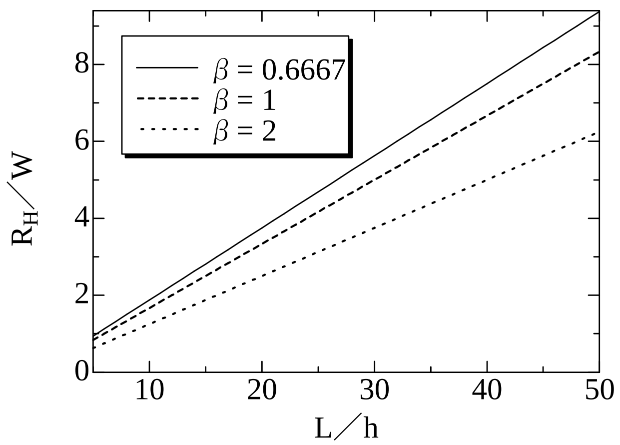

Figure 6. Girder with fixed bearings at both ends

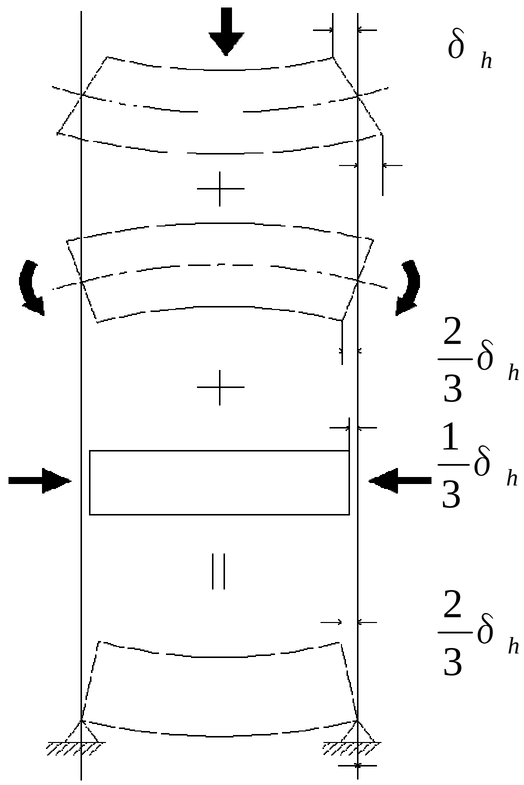

Figure 7. Restriction force at girder end

Figure 7 shows the relationship between L/h (= α) and the binding force R_H. The horizontal displacement at the upper end of the girder is obtained as follows.

$$\delta_{UP} = \delta_h - \delta_{MRH} + \Delta L = 2\, \beta\, \delta_h / (\beta+2) \tag{13}$$

When α = 20 and β = 1, horizontal displacements δ_{MRH} = (2/3)δh, and Δ_L = (1/3)δ_h and δ_{UP} = = (2/3)δ_h are obtained from Eq. (12) and (13), respectively. The binding force R_H is equal to 3.333W. Thus, when a moving load of 100 kN acts over the girder, a binding force of 333 kN acts at the bearing along the bridge axis. Given the magnitude and repetitive nature of this loading, deterioration of such bearings and their substructures is practically inevitable.

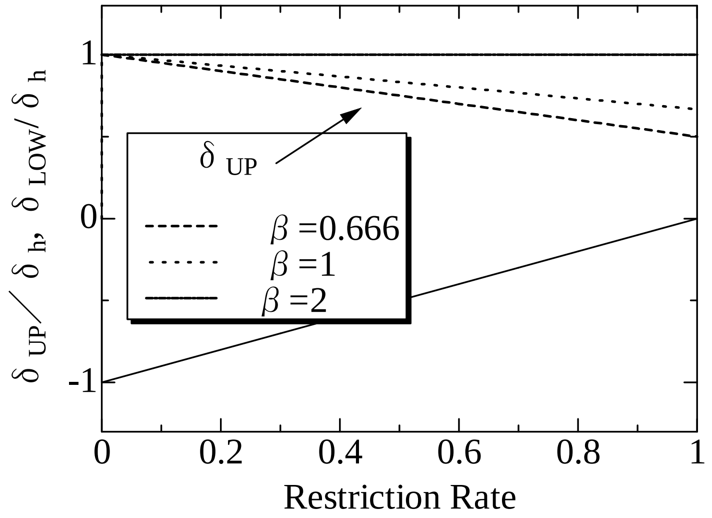

Figure 8. Horizontal deflection ratio at upper and lower girder end.

In the field, the soundness diagnostic of bridge can be evaluated more fully if we extrapolate the generated binding force acting upon a bearing from the amount of the horizontal displacement observed at a girder's ends. The binding force R and the horizontal displacement δ_{UP} and δ_{LOW} at the upper and lower end of a girder are respectively expressed using γ, as follows. γ indicates the ratio of the restraint.

$$R = \gamma R_H = \alpha \gamma W / \{ 2(\beta+2)\} \tag{14}$$

$$\delta_{UP} = \delta_h - \delta_{MRH} + L = \delta_h \{1+ \gamma (\beta - 2)/ (\beta+2)\} \tag{15}$$

$$\delta_{LOW} = -\delta_h + \delta_{MRH} + \Delta L = -\delta_h (1- \gamma) \tag{16}$$

Figure 8 shows the horizontal displacements at the upper and lower en ds of a girder with respect to the ratio of the restraint for values ranging from −1 to 1. The horizontal displacement δUP is plotted for β values of 2/3 (= 0.666), 1, and 2.

As shown, the horizontal displacement δUP remains constant when β = 2. Thus, the ratio δ_{UP}/δ_h is constant with the ratio γ of the restraint because the magnitudes of the horizontal displacement δ_{MRH} generated by bending moment M_{RH} and the axial displacement caused by the binding force R_H are equal and in opposite directions, so the absolute values of the displacements are equal. The horizontal displacement δ_{LOW} can be used as an indicator to judge the soundness of a bearing if the substructure is absolutely fixed because the displacement δ_{LOW} depends on the ratio β.

5. Measurement of horizontal displacement

The amount of deflection at a span center under live loading generally ranges from several millimeters to several centimeters. Therefore, the amount of horizontal displacement at a girder end is estimated to range from several hundred micrometers to several millimeters. To measure dynamic horizontal displacements accurately under live loading conditions such as during the passage of a train, high-precision displacement sensors that have a resolution of several micrometers are required. Many devices with sufficient precision are currently available for making such measurements, and specifications for the sensor used for our measurements are shown in table 1.

Table 1. The specifications of the sensor

Resolution capacity	~0.4 μm
Measurement range	0÷10 mm
Responsiveness	8 kHz, 18 KHz
Waterproof performance	IP67

This sensor measures displacement using eddy currents, and high accuracy is possible because the sensor is a non-contact type that is immune to errors typically encountered when measuring contact pressure. Moreover, this sensor responds precisely to the low frequency vibrations that occur in bridges.

A sensor fixed at one end of one girder with a magnetic stand measures the distance from a steel fixture at the other end without contact, as shown in Figure 9 and 10.

Figure 9. Installed sensors at girder ends

Figure 10. Sensors for measuring girder deflection

Relative horizontal displacements between the upper and lower girder ends on the 1st pier were observed before bearing replacement was carried out, and a graph of the dynamic displacements is shown in Figure 11(a). The relative displacement observed between the lower ends is far smaller than that of the upper ends of the girders, indicating that the bearings are restrained and are preventing the horizontal deformations that would be generated by passing trains were the bearings in good condition. This situation is the result of deterioration in the sliding function of these bearings, due to age. Figure 11(b) shows the observed displacements at the upper and lower positions of the girder ends on the 2nd pier after bearing replacement, when a train passed along the bridge. It was confirmed that the replaced bearings provided the desired sliding function after the repair.

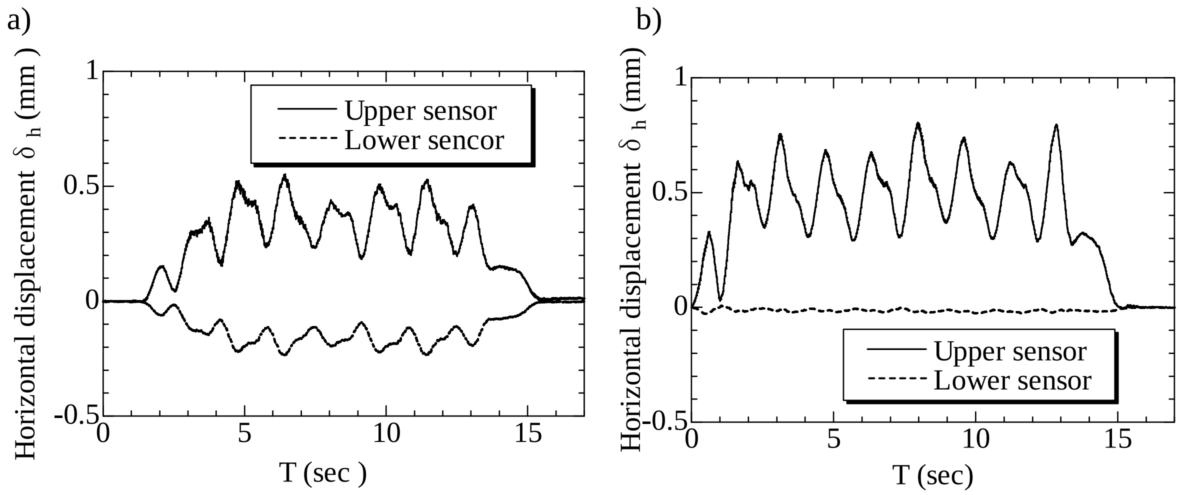

Figure 11. Obserbed horizontal displacement(a) before bearing replacement
(b) after bearing replacement

6. Conclusion

In general, vertical deflections are measured to enable evaluation of a bridge structure's soundness, but directly measuring such deflections is often difficult.

We showed that the vertical deflection of a girder can be calculated by measuring the horizontal displacement generated at the upper and lower ends of bridge girders. Furthermore, the horizontal binding force caused by deterioration in bearing function can be estimated by measuring the horizontal displacement generated at girder ends.

Girder ends are displaced to a degree approximately 1/10th that of the span center of a girder under live loads, so a high-precision displacement sensor is required to accurately measure dynamic horizontal displacements. We verified that horizontal displacements can be accurately measured using a precision displacement sensor, through the results of measurements in the field during the course of maintenance work.

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